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WORKING PAPER N° 2006 - 51

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JEL Codes: D43, D82, G22

Keywords: Non-exclusivity, insurance, moral hazard

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November 2006

Abstract

We study an economy where intermediaries compete over contracts in a nonexclusive insurance market affected by moral hazard. Our setting is the same as that developed in Bisin and Guaitoli [2004]. The present note provides a counterexample to the set of necessary conditions for high effort equilibria developed in Bisin and Guaitoli [2004] and suggests an alternative equilibrium characterization.

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JEL Classification: D43, D82, G22.

In a recent paper, Bisin and Guaitoli [2004] (BG) investigate competition among intermediaries in nonexclusive markets affected by moral hazard. The work introduces necessary conditions to be verified at any (high effort) equilibrium and emphasizes the role of latent contracts. Latent contracts are offers that are not bought at equilibrium: their role is to threat any single intermediary from increasing his market share by reducing his price. When these offers are issued, market equilibria exhibit non-competitive features, as positive profit for intermediaries and underinsurance for consumers, relatively to the standard scenario of competition under exclusivity. However, BG obtain a constrained efficiency result: whenever a social planner is *not* allowed to enforce exclusivity of trades, then every equilibrium allocation will be efficient from the point of view of such a planner (third best efficiency).

The present note argues that the BG analysis is incomplete, and that the conditions they introduce are therefore not necessary. To this extent, we suggest new insights both at a positive and at a normative level. We identify an additional set of equilibrium allocations which are supported by only one active intermediary, earning a positive profit. The amount of latent insurance needed to support this sort of allocations is higher than what conjectured by BG (and the corresponding price is lower). Interestingly, these situations give rise to to an insufficient provision of insurance from the only existing intermediary: given the informational constraints and the impossibility to control trades, there is room for a social planner to improve agents' welfare.

We consider the same setting as BG. There is an insurance economy lasting two periods. It is populated by a single representative consumer and by a countably infinite set of intermediaries. The probability distribution over the set of idiosyncratic states $\{1, 2\}$ depends on an unobservable effort $e = \{a, b\}$. Uncertainty affects the consumer's endowment $w = (w_1, w_2) \in \mathbb{R}^2_+$, with $w_2 > w_1$: $\pi_a(\pi_b)$ is taken to be the probability of occurrence of state 2 if $a(b)$ is chosen, with $a > b$ and $\pi_a > \pi_b$.

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We thank Alberto Bisin for his extremely useful comments on a previous draft of this work. We are also grateful to G. Bloise, E. Campioni, M. Hellwig, G.Piaser, U. Rajan and to seminar participants at Universitat de Rosario, at the JMA meeting (Nantes), the 2006 SED conference (Bodrum), and the 2006 EGRIE conference (Barcelona). All responsibility remains to the authors.

Trades are represented by insurance contracts offered by intermediaries to the single consumer. Every intermediary $i \in N$ offers a contract $d^i = (d_1^i, d_2^i)$ consisting in a pair of state-contingent transfers. The agent could buy a fraction λ_i of this contract. In addition, every $i-th$ intermediary decides whether Λ_i (the set of admissible λ_i) is $[0,1]$ (divisible) or $\{0,1\}$ (indivisible).[1] Insurance relationships are nonexclusive: the consumer chooses a subset of intermediaries to trade with and her decisions cannot be contracted upon.

The payoff to intermediary i is given by $V^i = -(\pi_e d_2^i + (1-\pi_e)d_1^i)\lambda_i$, when the effort e is chosen and the fraction λ_i of the offer d^i is bought. We denote $\vec{\pi}_e = ((1-\pi_e), \pi_e)$, and we then write $V^i = -\vec{\pi}_e \cdot d^i \lambda_i$. The agent-consumer is risk-averse. Her utility from consuming c_s, with $s = \{1,2\}$, is given by $u : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, which is continuous, increasing and concave. The corresponding expected utility is $\tilde{U}(c_1, c_2, e) = (1-\pi_e)u(c_1) + \pi_e u(c_2) - e$, where e denotes the cost of effort and $c_s = w_s + \sum_{i \in J} \lambda_i d^i$ is the contingent consumption. In what follows, we will always refer to $U(C) = \tilde{U}(C, e(C))$, with $e(C) \in \arg\max_e \tilde{U}(C,e)$. We also take $\mathcal{A} = \{C \in \mathbb{R}_+^2 \;/\; e(C) = a\}$ to be the set of ex-post consumption profiles inducing the choice $e = a$ and $\mathcal{B} = \{C \in \mathbb{R}_+^2 \;/\; e(C) = b\}$ to be that inducing $e = b$.

We will henceforth take the consumer's utility to be $u(c) = c^\gamma$ and $\gamma < 1$. Consider the following players' behaviors:

i) $d^1 = C - W, \quad \Lambda^1 = \{0,1\}$,

ii) $d^2 = d^3 = L - C, \quad \Lambda^2 = \Lambda^3 = [0,1]$,

iii) $d^4 = d^5 = \cdots = d^N = (0,0)$,

iv) $\lambda_1 = 1, \lambda_2 = \lambda_2 = \cdots = \lambda_N = 0$, and $e = a$,

with $C = (c_1, c_2) \in \mathcal{A}$, $L = (l_1, l_2) \in \mathcal{B}$, $c_1 > w_1$, and $c_2 < w_2$. That is, intermediary 1 makes a positive insurance, take-it or leave-it offer (indivisible contract), while intermediaries 2 and 3 propose a divisible contract, and all other intermediaries offer the null contract $(0,0)$. The consumer accepts the offer of intermediary 1 and rejects those of intermediaries 2 and 3, as well as the remaining ones. In addition, she selects the high effort.

Figure 1 depicts the consumer's feasible consumption set given the endowment W and the offers' array $\left(d^1, d^2, \cdots, d^N\right)$.

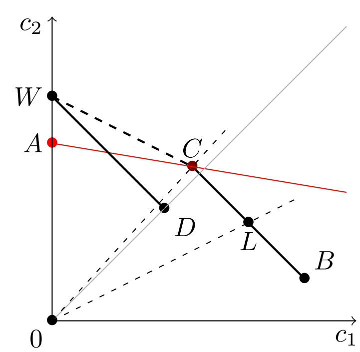

Figure 1: Set of feasible consumption allocations

The array (C, L) is identified by the following:

[1] The case of indivisibility clearly denotes a situation where the $i-th$ intermediary is making a take-it or leave-it offer.

$$U(C) = U(L) \tag{1}$$

$$\tau^L c_1 + c_2 = \tau^L l_1 + l_2 \tag{2}$$

$$\pi_a(1-\pi_a)\big(u'(c_1) - u'(c_2)\big) = \pi_a\,(1-\pi_b)\,u'(l_1) - \pi_b\,(1-\pi_a)\,u'(l_2), \tag{3}$$

and it satisfies the additional conditions

$$\frac{(1-\pi_b)}{\pi_b} > \tau^L > \tau^C > |\frac{w_2 - c_2}{w_1 - c_1}| > \frac{(1-\pi_a)}{\pi_a}, \tag{4}$$

$$w_1 + 2(l_1 - c_1) = w_2 + 2(l_2 - c_2), \tag{5}$$

$$\frac{w_2 + 3(l_2 - c_2)}{w_1 + 3(l_1 - c_1)} < \frac{l_2}{l_1}, \qquad -\vec{\pi}_a \cdot \big(L - C\big) < -\vec{\pi}_a \cdot \big(C - W\big), \tag{6}$$

where $\tau^L = \frac{(1-\pi_b)\,u'(l_1)}{\pi_b u'(l_2)}$ and $\tau^C = \frac{(1-\pi_a)\,u'(c_1)}{\pi_a u'(c_2)}$ denote the marginal rate of substitution of the single consumer evaluated at L and at C, respectively.

Relationships $(1)-(6)$ have a straightforward interpretation: (1) requires the single agent to be indifferent between C and L, and (2) states that the price of the contracts offered by intermediaries 2 and 3 is τ^L. Condition (3), which was not identified by BG, plays a key role in our analysis: it guarantees that every (small) deviation associated to a reduction in the agent's payoff induces $e = b$ as the optimal effort choice. The first inequality in (4) states that the price of contracts d^2 and d^3 is (strictly) smaller than the fair price under low effort; the second and the third one guarantee that accepting the offer $d^1 = C - W$ is an optimal choice for the consumer when she selects $e = a$. The last inequality implies that the only active intermediary earns a positive profit offering d^1, at a price $|\frac{w_2 - c_2}{w_1 - c_1}|$. Conditions (5) and (6) guarantee that the consumer's threat of selecting the low effort will still be effective even when large deviations are considered.

Summarizing: given W and the offers' array $\big(d^1, d^2, ..., d^N\big)$, the agent can achieve the allocation C as well as every element on the line of slope τ^L connecting C and L (see Figure 1). The availability of the allocation L is hence due to the latent contracts d^2 and d^3 that the agent stands ready to buy whenever the low effort $e = b$ is selected. Every array (C, L) satisfying $(1)-(6)$ is such that the corresponding latent contracts earn a negative profit if they were accepted. The main contribution of this note is to establish the following:

Proposition 1 *Consider an economy where consumer's preferences are represented by* $u(c) = c^\gamma$. *Then, for every array* (C, L) *satisfying* $(1)-(6)$, *the allocation* $C \in \mathcal{A}$ *can be supported as a pure strategy equilibrium by the players' behaviors described in* $(i)-(iv)$.

One should notice that we are explicitly introducing both necessary conditions ((1) to (3)) and sufficient ones ((4) to (6)). This allows us to fully analyze all possible deviations from active and inactive intermediaries. In the following, we first show that the set of allocations (C, L) satisfying $(1)-(6)$ is non-empty. Then, we stress the restrictions induced by these conditions on the consumer's optimal behavior. Finally, we argue that the described players' behaviors form an equilibrium. All Proofs are collected in the Appendix. As a starting point, we have:

Lemma 1 *If* $u(c) = c^\gamma$, *there exists an array of parameters* $\big(W, \gamma, \pi_a, \pi_b, a, b\big)$ *such that the system*

$(1)-(6)$ *has a solution* $\big(C, L\big)$, *with* $C \in \mathcal{A}$ *and* $L \in \mathcal{B}$.

We now analyze the consumer's behavior, emphasizing those features of her choices which are relevant to analyze the deviation stage. To this extent, we refer to the allocation $C_\alpha \in \mathcal{A}$. For every $\alpha \in \mathbb{R}$, C_α is taken to be at the intersection between the (iso-profit) line of slope $\frac{1-\pi_a}{\pi_a}$ passing through C and the line connecting αC and αL: every C_α hence guarantees the same aggregate profit as that earned by the single intermediary in in C. That is: $C_\alpha = (c_1 + \pi_a f(\alpha), c_2 - (1-\pi_a) f(\alpha))$, with $f'(\alpha) > 0$.

Lemma 2 *If* $u(c) = c^\gamma$ *and the allocations* C *and* L *satisfy* $(1)-(6)$*, then:*

1. $U(C_\alpha) < U(\alpha L) \quad \forall \alpha \neq 1$ *such that* $C_\alpha \in \mathcal{A}$,
2. $U(K) < U(\alpha L) \quad \forall K \in \mathcal{A}$ *lying between* C_α *and* αL,
3. $U(\alpha C) < U(\beta L) \quad \forall (\alpha, \beta)$ *with* $\beta > \alpha > 1$ *and the line connecting* αC *and* C_β *has slope* τ^C.

To complete the proof of Proposition 1 we have to show that the allocation $C \in \mathcal{A}$ can be supported at equilibrium.

Lemma 3 *If* $u(c) = c^\gamma$*, then the players' behaviors described in* $(i)-(iv)$ *constitute a subgame perfect equilibrium of the game played amongst the* N *intermediaries in the presence of a single agent. The corresponding equilibrium allocation will be* $C \in \mathcal{A}$.

We now relate in greater detail our example to the BG work. BG present their main result in Proposition 2, which is stated as follows:[2]
If the set of consumption allocations $X = \{(x_1, x_2)/(1-\pi_a)u(x_1) + \pi_a u(x_2) - u\big((1-\pi_b)x_1 + \pi_b(x_2)\big) - (a-b) \geq 0\}$ *is non empty, then any pure strategy equilibrium with* $e = a$ *induces an allocation* $C = (c_1, c_2)$ *such that:*

$$(1-\pi_a)u(c_1) + \pi_a u(c_2) - u\big((1-\pi_b)c_1 + \pi_b(c_2)\big) - (a-b) = 0. \tag{7}$$

Equation (7) can be interpreted as an indifference condition for the agent between the equilibrium allocation C where $e = a$ is chosen, and another feasible allocation on the 45 degree line, that can be reached buying additional insurance at the fair price $\frac{1-\pi_b}{\pi_b}$. In addition, their Proposition 3 states that at any high effort equilibrium there will be at least two active intermediaries,[3] and Proposition 4 provides a constrained efficiency result: every high effort equilibrium is third best optimal.[4] In particular, the feasible set for the social planner in a "third best program" is given by the set X, and any third best allocation belongs to its frontier.
The equilibrium we have constructed does not exhibit the properties described by BG. To clarify this, we first remark that if conditions $(1)-(6)$ are satisfied, then (7) will never hold as an equality. This directly contradicts BG Proposition 2.[5]

[2] See Bisin and Guaitoli [2004], p. 314.

[3] See Bisin and Guaitoli [2004], p. 315.

[4] See Bisin and Guaitoli [2004], p. 319. For a formal definition of the notion of third best optimality see also Kahn and Mookherjee [1998].

[5] To show that we can in principle apply BG Proposition 2, one has to consider the point $F = (f, f)$ on the 45 degree line, with $f = \big(\frac{\tau^L l_1 + l_2}{\tau^L + 1}\big)$. F belongs to the tangent to the consumer's indifference curve at L, and $U(L) > U(F)$, since $F \in \mathcal{B}$. Using (1) and (2), the last inequality can be written as: $(1-\pi_a)u(c_1) + \pi_a u(c_2) - (a-b) > u\big(\frac{\tau^L c_1 + c_2}{\tau^L + 1}\big)$. Also, since $\tau^L < \frac{1-\pi_b}{\pi_b}$, we get: $\frac{\tau^L c_1 + c_2}{\tau^L + 1} > \frac{\frac{1-\pi_b}{\pi_b} c_1 + c_2}{\frac{1-\pi_b}{\pi_b} + 1} = (1-\pi_b)c_1 + \pi_b c_2$. We can hence conclude that $(1-\pi_a)u(c_1) + \pi_a u(c_2) - (a-b) > u((1-\pi_b)c_1 + \pi_b c_2)$, guaranteeing that the set X introduced in BG is non-empty.

In addition, our equilibrium is supported by only one active intermediary and this indeed also contradicts BG Proposition 3.[6]

One should also notice that the equilibrium allocation we exhibit will not be on the frontier of the set X, which guarantees that it fails to be third best efficient, contradicting BG's Proposition 4.

A salient feature of our example is that every latent contract implies *negative* (latent) profits for the issuer; that is, its price is lower than the fair price under low effort. The result contrasts BG's conjecture that any pure strategy equilibrium with negative latent profits would never be robust to deviations from latent intermediaries. The problem in their proof appears when they argue that every deviation to a negative insurance contract by any latent intermediary would induce the effort choice $e = b$.[7] We have shown in contrast that, in such a situation, the consumer will always take the opportunity of the negative insurance contract to reach a better allocation in $\mathcal{A}$, which makes the deviation unprofitable.

Moreover, one can show that there exists a continuum of points in a neighborhood of (C, L) that can be also supported at a pure strategy equilibrium.

Finally, we closely discuss the relevance of situations where intermediaries earn negative latent profits. We remark here that any allocation $C = (c_1, c_2)$ of our Proposition 1 can be as well supported as a symmetric equilibrium of a simpler game where two competitors (principals) can offer any possible subset of alternatives (menus) to the single agent.[8]This provides a further rationale for this sort of equilibria. Instead of thinking of (latent) intermediaries who are offering their contracts anticipating that they will not be accepted and that they will (eventually) incur a loss, one can represent latent offers as a part of a nonlinear menu proposed by a single principal. In this last case, every principal issues these additional offers to strategically protect his own rents from his rivals' opportunistic behaviors. Importantly, equilibria of this sort have been intensively examined in the recent literature on competing mechanism games. The possibility to sustain outcomes through offers which are not accepted at equilibrium but are strategically issued by competitors is indeed at the root of the failure of the Revelation Principle in games with multiple principals.[9]

Our analysis therefore suggests that equilibria can be Pareto-ranked according to the price of latent contracts. In particular, those allocations (if there is any) supported by latent insurance offered at a fair price under low effort turn out to be (constrained) efficient. Thus, even though there is no general argument to get rid of this sort of outcomes,[10] it is remarkable that a social planner would never have an incentive to select them.

Appendix

- Proof of Lemma 1

We first fix $\gamma = 1/2,\ C = (25, 50)$ and $\tau^L = 1$. Then, we consider all the solutions such that $w_1 = 0$ and

[6]More precisely, we are showing that the existence of two distinct allocations $Y \in \mathcal{A}$ and $Z \in \mathcal{A}$ such that $U(Y) = U(Z)$ is *not* a necessary condition for getting a high effort equilibrium, contrarily to what BG have argued (see Bisin and Guaitoli [2004], p. 326).

[7]See Bisin and Guaitoli [2004], p. 326.

[8]An extensive analysis of the relationship between latent contracts in competitive economies and menus equilibria is presented in Attar et al. [2007]. In particular, it is shown that the allocation $C = (c_1, c_2)$ can be supported at equilibrium in a game with two principals, each offering the (convex) menu $\mathcal{M}^i = \{\mu^i \frac{W+C}{2} + \lambda^i d^2\}$, for $i \in \{1, 2\}$. The agent selects her preferred element in $\mathcal{M}^i$ by choosing μ^i and $\lambda^i \in [0, 1]$. The choice $\mu^i = 1$ and $\lambda^i = 0$ turns out to be an equilibrium behavior for the single agent.

[9]See, among many others, Martimort and Stole [2002].

[10]The standard notion of trembling hand perfection cannot be straightforwardly extended to games with continuous decision sets.

$w_2 = +(\tau^C - 0.01)c_1 + c_2$. For (3) and (6) to be verified, one has to impose $l_1 = c_1 + \frac{(\tau^C - 0.01)\,c_1 + c_2}{2(1+\tau^L)}$ and $l_2 = c_2 - \tau^L \frac{(\tau^C - 0.01)\,c_1 + c_2}{2(1+\tau^L)}$. The corresponding values of π_a and π_b are identified by the relationships $MRS_C = \tau^C$ and $MRS_L = \tau^L$: $\pi_a = \frac{\sqrt{c_2}}{\tau^C\sqrt{c_1}+\sqrt{c_2}}$, $\pi_b = \frac{\sqrt{l_2}}{\tau^L\sqrt{l_1}+\sqrt{L_2}}$. Then, (4) can be rewritten as:

$$\tau^c\left(\sqrt{l_1}+\sqrt{l_2}\right)\left(\sqrt{c_2}-\sqrt{c_1}\right) = c_2 - (\tau^C)^2 c_1,$$

where we used the fact that $\tau^L = 1$. This equation has two solutions in τ^C, a positive and a negative one. We select the positive one so to get $\tau^c \approx 0.996291$.[11] Given τ^C, from (6) we derive: $L \approx (43.6643, 31.3357)$, and one has to notice that since $l_1 > l_2$, we directly have that $L \in \mathcal{B}$. Finally, the parameter $a-b$ is chosen so to satisfy (2). In particular, we get $a - b \approx 0.140827$. It is then possible to verify by direct inspection that all inequalities in (5) and (7) are verified and that $C \in \mathcal{A}$.[12]

- Proof of Lemma 2

1. If $C_\alpha \in \mathcal{A}$, then $\frac{d}{d\alpha}U(C_\alpha) = \pi_a(1-\pi_a)f'(\alpha)\big(u'(c_{1\alpha}) - u'(c_{2\alpha})\big)$. One should notice that for $\alpha \geq (\leq)1$, $\frac{d}{d\alpha}U(C_\alpha)$ is bounded above (below) by $\pi_a\,(1-\pi_a)\alpha^{\gamma-1}f'(\alpha)\big(u'(c_1) - u'(c_2)\big)$, since $u'(c_{1\alpha})$ is lower (greater) than $u'(\alpha c_1)$. Furthermore:

$$\frac{d}{d\alpha}U(\alpha L) = \alpha^{1-\gamma}f'(\alpha)\Big(\pi_a\,(1-\pi_b)\,u'(l_1) - (1-\pi_a)\,\pi_b\,u'(l_2)\Big)$$

   Then, for $\alpha \geq 1$ ($\alpha \leq 1$), $\frac{d}{d\alpha}\big[U(C_\alpha) - U(\alpha L)\big]$ is bounded above (below) by $\alpha^{\gamma-1}f'(\alpha)\Big(\pi_a(1-\pi_a)\big(u'(c_1) - u'(c_2)\big) - \pi_a\,(1-\pi_b)\,u'(l_1) - (1-\pi_a)\,\pi_b\,u'(l_2)\Big)$ which is zero by (3). Hence, $U(C_\alpha) - U(\alpha L)$ has a maximum in $\alpha = 1$. If $\alpha = 1$, $U(C_\alpha) - U(\alpha L) = U(C) - U(L) = 0$ from (1).

2. Let us first take α_0 to be the value of α such that $MRS|C_{\alpha_0} = \tau^L$. For $\alpha \geq \alpha_0$, one can directly verify that $MRS|_{C_\alpha} \leq \tau^L$. It is then immediate that $U(K) < U(C_\alpha)$ for every $K \in \mathcal{A}$ belonging to the segment (of slope τ^L) that connects C_α and αL. It follows from part 1 of the lemma that $U(K) < U(\alpha L)$. For $\alpha < \alpha_0$, $MRS|_{C_\alpha} > \tau^L$, and the agent's consumption choice on the line passing through C_α and αL will be $\frac{\alpha}{\alpha_0}C_{\alpha_0} \in \mathcal{A}$. Using again point 1 of the lemma we get, after some computations:

$$U(K) - U(\alpha L) \leq U\big(\frac{\alpha}{\alpha_0}C_{\alpha_0}\big) - U(\alpha L) = \big(\frac{\alpha}{\alpha_0}\big)^\gamma\big(U(C_{\alpha_0}) - U(\alpha_0 L)\big) + (a-b)\big((\frac{\alpha}{\alpha_0})^\gamma - 1\big) < 0$$

3. We denote $\tau^a = \frac{1-\pi_a}{\pi_a}$. Given the definition of C_β and recalling that αC and C_β are on a line of slope τ^C:

$$\alpha(\tau^C c_1 + c_2) = (\tau^C c_1 + c_2) + f(\beta)\pi_a(\tau^C - \tau^a) \Leftrightarrow \frac{\alpha - 1}{\pi_a f(\beta)} = \frac{\tau^C - \tau^a}{\tau^C c_1 + c_2}. \tag{8}$$

   Now, since also C_β and βL are on a line of slope τ^L:

$$\beta(\tau^L l_1 + l_2) = (\tau^L c_1 + c_2) + f(\beta)\pi_a(\tau^l - \tau^a) \Leftrightarrow \frac{\beta - 1}{\pi_a f(\beta)} = \frac{\tau^L - \tau^a}{\tau^L c_1 + c_2}, \tag{9}$$

   where we used the fact that $\tau^L l_1 + l_2 = \tau^L c_1 + c_2$ from (2). Then, consider the difference:

$$U(\beta L) - U(\alpha C) = [(1-\pi_b)\,u(\beta l_1) + \pi_b\,u(\beta l_2) - b)] - [(1-\pi_a)\,u(\alpha c_1) + \pi_a\,u(\alpha c_2) - a)]\,,$$

   which can be rewritten as $g(\beta)$, given (8). Differentiating:

[11] The solution has been calculated using the *NSolve* function in *Mathematica*. The code is available from the authors.
[12] This has been established running a standard test in *Mathematica*.

$$
\begin{aligned}
g'(\beta) &= \left[(1-\pi_b)\, u'(l_1)l_1 + \pi_b\, u'(l_2)l_2\right]\beta^{\gamma-1} - \left[(1-\pi_a)\, u'(c_1)c_1 + \pi_a u'(c_2)c_2\right]\alpha^{\gamma-1}\frac{\tau^C-\tau^a}{\tau^L-\tau^a} \\
&= \frac{1}{\tau^L-\tau^a}\left[\pi_b u'(l_2)(\tau^L-\tau^a)(\tau^L l_1+l_2)\beta^{\gamma-1} - \pi_a u'(c_2)(\tau^C-\tau^a)(\tau^C c_1+c_2)\alpha^{\gamma-1}\right]
\end{aligned}
$$

We also remark that (3) can be rewritten as $\pi_b\ u'(l_2)[\tau^L-\tau^a] = \pi_a\ u'(c_2)[\tau^C-\tau^a]$. Since $\beta > \alpha$: $\left(\beta^{\gamma-1}(\tau^L l_1+l_2) - \alpha^{\gamma-1}(\tau^C l_1+l_2)\right) > \left(\tau^L-\tau^C\right)l_1$. Concluding, $g'(\beta) > 0$ and as $g(0)=0$, we have that $g(\beta) > 0$.

- Proof of Lemma 3

Let us first examine the agent's behavior. Then, $C = (c_1, c_2)$ is (weakly) preferred to any of the allocations belonging to the frontier of the feasible (consumption) set. It will hence be a best reply for the agent to accept the contract of intermediary 1 and reject all the others.

Then, we consider intermediaries'. It is sufficient to restrict the analysis to deviations in take-it or leave-it offers. Let us consider intermediary 1. We first examine the deviations $d1\prime$ inducing a high level of effort and we take $K \in \mathcal{A}$ to be the optimal consumption choice of the agent at the deviation stage. If $d^{1\prime}$ is a negative insurance contract, to guarantee positive profits, its price should be below $\frac{1-\pi_a}{\pi_a}$, since the agent will select $e = a$. Then, whenever such a $d^{1\prime}$ is bought, the corresponding K should lie to the left of the line connecting W and D (see Figure 1), which contradicts the fact that K is optimally chosen.
If $d^{1\prime}$ is a positive insurance contract, to guarantee positive profits it must be that $-\vec{\pi}_a \cdot d^{1\prime} > -\vec{\pi}_a \cdot d^1$. It hence follows from (6) that $-\vec{\pi}_a \cdot d^{1\prime} > -\vec{\pi}_a \cdot d^2$. Moreover, $d^{1\prime}$ cannot be offered at a price (strictly) higher than τ^L,[13] and this implies $d^{1\prime} = d^2 + d^{1\prime\prime}$, with $d^{1\prime\prime}$ being a positive insurance contract which price is lower than τ^L. The terminal point on the frontier of feasible consumptions corresponding to the offers $(d^{1\prime}, d^2, ..., d^N)$, i.e. $W + d^{1\prime\prime} + 3d^2$, [14], will hence fall below the line passing through the origin and L, as it is the case for the point $W + 3d^2$.[15] At the deviation stage, the agent can therefore achieve her optimal choice in the subset $\mathcal{B}$, since it lies on the line passing through the origin and L. Thus, if one takes the particular $\alpha \in \mathbb{R}_+$ such that C_α, K and αL are on the same line, then $K \in \mathcal{A}$ will be between C_α and αL. Hence, lemma 2 implies that $U(K) < U(\alpha L)$, that constitutes a contradiction, since αL is available.[16]
Intermediary 1 cannot profitably deviate inducing low effort either. A deviation to a positive insurance contract inducing $e = b$ could be profitable only if the price is higher than $\frac{1-\pi_b}{\pi_b}$. The agent, though, will never have any incentive to buy such a contract since she the optimal available allocation when $d^{1\prime}$ is refused is a point like D on the 45 degree line (see Figure 1). If $d^{1\prime}$ is a negative insurance contract, the relevant price should be higher than τ^L, for the agent to have an incentive to buy it, and lower than $\frac{(1-\pi_b)}{\pi_b}$, to be profitable for the deviator. Since the agent can always achieve the allocation D by rejecting $d^{1\prime}$, the deviating contract will never be bought.
We now consider the behavior of intermediary 2 (and 3) and first show that there is no profitable deviation inducing $e = a$. We keep denoting $K \in \mathcal{A}$ the optimal consumption choice of the agent, given the deviation $d^{2\prime}$ of intermediary 2. If $d^{2\prime}$ is a positive insurance contract,[17] and $-\vec{\pi}_a \cdot (K - C) > 0$, one can use an argument similar to that suggested in the previous paragraph. In this case, the relevant α will be greater than 1, which implies that the corresponding αL will be available and that lemma 2 can be applied.

[13] Otherwise, it would not be accepted by the agent.
[14] We recall that $d^2 + d^3 + d^2 = 3d^2$.
[15] See (6).
[16] If we now consider the optimal choice of the agent in the set $\mathcal{A}$, then it should be that $-\vec{\pi}_a \cdot (K - W) > -\vec{\pi}_a \cdot (C - W)$. That is, K will fall below the line of slope $\frac{1-\pi_a}{\pi_a}$ passing through C.
[17] The case of $d^{2\prime}$ being a negative insurance contract is not of great interest, since the agent will never have an incentive to accept a negative insurance contract issued at a price lower than $\frac{1-\pi_a}{\pi_a}$ and select the high effort $e = a$

The case $-\vec{\pi}_a \cdot (K - C) \leq 0$ only takes place when the contract d^1 is not bought at the deviation stage.[18] All feasible consumption choices are depicted in Figure 2.[19] For every $K \in \mathcal{A}$, the agent can always get an allocation like γL that is on the ray connecting the origin to L.[20] Let us now take C_β to be the intersection between the line of slope $\frac{1-\pi_a}{\pi_a}$ passing through C and the line connecting K and $M = K + d^1$. In addition, we denote βL (with $\beta < \gamma$) the projection of C_β through a line of slope τ^L on the ray connecting 0 and L. Finally, we let $\alpha C \in \mathcal{A}$ be the optimal choice of the agent along the line of slope τ^C passing through C_β. For every $K \in \mathcal{A}$ one has that:

$$U(K) \leq U(\alpha C) \tag{10}$$

$$U(\beta L) \leq U(\gamma L) \tag{11}$$

One should observe that (10) is satisfied by construction, since $|\frac{d_1^1}{d_2^1}| \leq \tau^C$. To establish (11) we remark that $\vec{\tau}^L \cdot K \leq \vec{\tau}^L \cdot M$, and, since C_β is between K and M, we have $\vec{\tau}^L \cdot C_\beta \leq \vec{\tau}^L \cdot M$, with $\vec{\tau}^L = \big(\tau^L, 1\big)$. That is, βL will fall at the left of γL, which gives (11). Hence, one can apply the last point of lemma 2 so to get $U(\alpha C) < U(\beta L)$, which, given (10) and (11), implies that $U(K) < U(\gamma L)$, a contradiction.

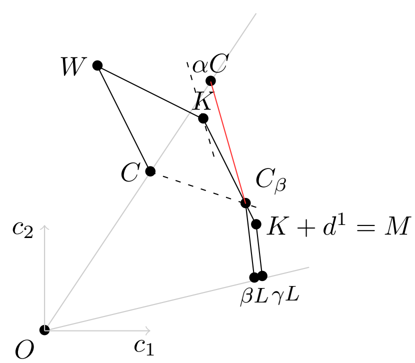

Figure 2: A deviation of intermediary 2

In a next step we investigate whether there is any deviation of intermediary 2 that induces the low effort $e = b$. If $d^{2\prime}$ is a positive insurance contract, then it must involve a price strictly higher than $\frac{1-\pi_b}{\pi_b}$ to be profitable. The fact that the consumer can already achieve her optimal choice at a price τ^L, guarantees that such a deviation would never be accepted. If $d^{2\prime}$ is a negative insurance contract, then its price should be (strictly) higher than τ^L. The frontier of the set of feasible consumptions will then be the line connecting $W + d^{2\prime}$, $C + d^{2\prime}$ and $L + d^{2\prime}$. Consider now the particular $\alpha > 1$ such that $C + d^{2\prime}$, αC and αL are on the same line. One should then notice that αL is available to the consumer, given that $C + d^{2\prime}$ falls over the ray connecting the origin with C (i.e. it is at the north-west of αC). One should then observe that $U(\alpha L) < U(\alpha C)$, that constitutes a contradiction.[21]

Finally, one has to consider the deviations of all intermediaries offering the null contract $(0, 0)$. To show that these deviations will never be profitable, one can use the same argument developed for the latent intermediaries 2 and 3. Indeed, when an intermediary offering a null contract (i.e. an entrant) deviates, the set of feasible allocations for the agent coincides with the set of allocations that were available following the deviation $d^{2\prime}$ together with some additional amount of insurance available at a price τ^L. The agent will hence behave in the same way following a deviation to $d^{2\prime}$ and following a deviation from any of the entrants.[22]

[18] Whenever d^1 is bought, then we have that: $-\vec{\pi}_a \cdot (K - W) \geq -\vec{\pi}_a \cdot d^1 - \vec{\pi}_a \cdot d^{2\prime} > -\vec{\pi}_a \cdot d^1 = -\vec{\pi}_a \cdot (C - W)$.

[19] Notice that K does not necessarily lie between C and αC.

[20] The availability of γL is guaranteed by construction. Indeed, we already know that adding d^3 to C is enough to achieve L. Since $d^{2\prime}$ is a positive insurance contract, by adding $d^{2\prime}$ and d^3 to C we make γL available.

[21] We have that $U(\alpha C) - U(\alpha L) = \big(U(\alpha C) + a\big) - \big(U(\alpha L) + b\big) - (a - b) = \alpha^\gamma \big(U(C) + a\big) - \alpha^\gamma \big(U(L) + b\big) - (a - b) = (\alpha^\gamma - 1)(a - b)$

[22] More precisely, when we considered deviations that induce $e = a$, it was enough to prove that the consumer always has an

References

Andrea Attar, Eloisa Campioni, Arnold Chassagnon, and Uday Rajan. Incentives and competition under moral hazard. Mimeo IDEI, Toulouse, 2007.

Alberto Bisin and Danilo Guaitoli. Moral hazard with non-exclusive contracts. *Rand Journal of Economics*, 2:306–328, 2004.

Charles M. Kahn and Dilip Mookherjee. Competition and incentives with nonexclusive contracts. *RAND Journal of Economics*, 29(3):443–465, Autumn 1998.

David Martimort and Lars A. Stole. The revelation and delegation principles in common agency games. *Econometrica*, 70(4):1659–1673, July 2002.

incentive to select an allocation on the ray connecting 0 and L. This will of course remain available in this context. All deviations inducing $e = b$ were instead blocked independently of the availability of the optimal consumption choice in $\mathcal{B}$. Clearly, such an optimal choice will not be modified by the availability of additional insurance at a price τ^L.